Marian Catholic High School
Calculus
Summer Math Problems

Name ____

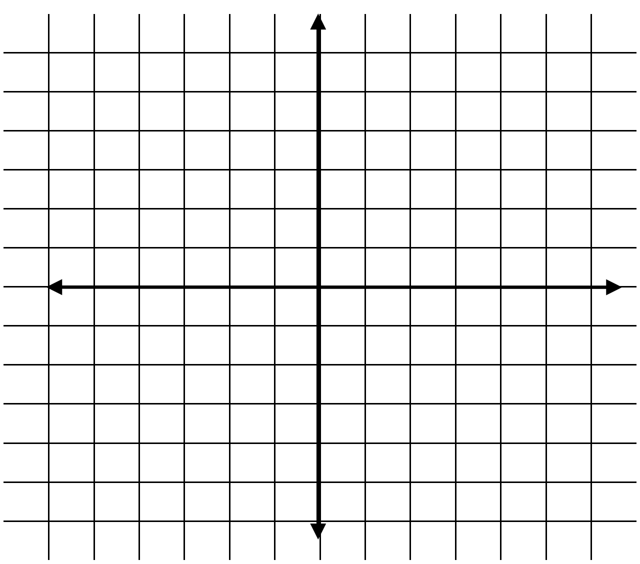
1. Evaluate the function at the given values of the independent variable and simplify.

   $f(x) = x^2 + 2x - 3$

   a. *f(-1)* **b.** *f(a)* **c.** *f(x + 1)*

2. Use the given conditions to write an equation of the line in slope-intercept form.
   Passing through (-2, -4) and (1, -1).

3. Begin by graphing the standard quadratic function, $f(x) = x^2$. Then use transformations to graph $f(x) = -(x-2)^2 + 1$.



4. Find the domain of the function.

$$f(x) = \frac{1}{x+7} + \frac{3}{x-9}$$

5. For the given function find an equation for the inverse function $f^{-1}(x)$.

$$f(x) = 3x - 1$$

6. Give the center and radius of the circle described by the equation.

$$(x-3)^2 + (y-1)^2 = 36$$

7. Find the zeros for the polynomial function.

$$f(x) = x^3 + 4x^2 + 4x$$

8. Solve the polynomial inequality.

$$6x^2 + x \geq 1$$

Solve each logarithmic equation.

9. $\log_3 x = 4$

10. $\log_6(x+5)+\log_6 x = 2$

Find each value by using a reference triangle or the unit circle. Show all work. Do not use a calculator.

11. $\sin\dfrac{3\pi}{4}$

12. $\tan\pi$

13. Determine the amplitude and period of the function. Then graph one period of the function. Label all tick marks.

$y = 2\cos 2x$

Find the exact value(s) of each function that are on $[0,2\pi]$. Do not use a calculator.

14. $\sin^{-1}\left(-\frac{1}{2}\right)$

15. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

16. Verify the identity.

$\tan x \csc x \cos x = 1$

Application Exercises

17. One yardstick for measuring how steadily-if slowly-athletic performance has improved is the mile run. In 1954, Roger Bannister of Britain cracked the 4-minute mark, setting the record for running a mile in 3 minutes, 59.4 seconds, or 239.4 seconds. In the half-century since then, the record has decreased by 0.3 second per year.

a. Express the record time for the mile run, *M*, as a function of the number of years after 1954, *x*.

b. If this trend continues, in which year will someone run a 3-minute. or 180- second mile?

18. A bird species in danger of extinction has a population that is decreasing exponentially $\left(A = A_0 e^{kt}\right)$. Five years ago the population was at 1400 and today only 1000 of the birds are alive. Once the population drops below 100, the situation will be irreversible. When will this happen?

19. People who believe in biorhythms claim that there are three cycles that rule our behavior-the physical, emotional, and mental. Each is a sine function of a certain period. The function for our emotional fluctuations is

$$E = \sin\frac{\pi}{4}t$$

where t is measured in days starting at birth. Emotional fluctuations, E, are measured from -1 to 1, inclusive, with 1 representing peak emotional well-being, -1 representing the low for emotional well being, and 0 representing feeling neither emotionally high nor low.

a. Find E corresponding to t = 7, 14, 21, 28, and 35. Describe what you observe.

b. What is the period for the emotional cycle?

Technology Exercises

20. Use a graphing utility with a viewing rectangle large enough to show end behavior of the graph of the polynomial function.

$$f(x) = x^3 + 13x^2 + 10x - 4$$

$X_{\min} =$ ________
$X_{\max} =$ ________
$X_{scl} =$ ________
$Y_{\min} =$ ________
$Y_{\max} =$ ________
$Y_{scl} =$ ________

21. Graph *f* and *g* in the same viewing rectangle n. Then describe the relationship of the graph of *g* to the graph of *f*.

$$f(xc) = \ln x, \quad g(x) = \ln(x+3)$$

22. Use a graphing utility to graph $y = \cos x$ and $y = 1 - \frac{x^2}{2} + \frac{x^4}{24}$ in a $\left[-\pi, \pi, \frac{\pi}{2}\right]$ by $[-2,2,1]$ viewing rectangle. How do the graphs compare?

Critical Thinking Exercises

23. *Freedom 7* was the spacecraft that carried the first American into space in 1961. Total flight time was 15 minutes and the spacecraft reached a maximum height of 116 miles. Consider a function, *s*, that expresses *Freedom 7's* height, *s(t)*, in miles, after *t* minutes. Is *s* a one-to one function? Explain your answer.

24. If $\log 3 = A$ and $\log 7 = B$, find $\log_7 9$ in terms of *A* and *B.*

25. Two buildings of equal height are 800 feet apart. An observer on the street between the buildings measures the angles of elevation to the tops of the buildings as 27° and 41°, respectively. How high, to the nearest foot, are the buildings?